Section 1-3.

4 Compute $\vec{a}\cdot(\vec{b}\times\vec{c})$, $\vec{a} = \vec{i} - 2\vec{j} + \vec{k}$, $\vec{b} = 2\vec{i} + \vec{j} + \vec{k}$.

Solution: $\vec{c} = 3\vec{i} - \vec{j} + 2\vec{k}$.

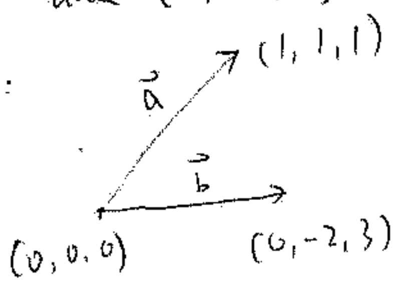
$$\vec{b}\times\vec{c} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 1 \\ 3 & -1 & 2 \end{vmatrix} = 3\vec{i} - \vec{j} - 5\vec{k}.$$

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = 3 + 2 - 5 = 0.$$

Rubric: 3 pts for cross product, 2pts for $\vec{a}\cdot(\vec{b}\times\vec{c})$.

6. Find the area of triangle with vertices $(0,0,0)$, $(1,1,1)$ and $(0,-2,3)$.

Solution:



$\text{Area} = \frac{1}{2}|\vec{a}\times\vec{b}|$

While
$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 0 & -2 & 3 \end{vmatrix} = 5\vec{i} - 3\vec{j} - 2\vec{k}.$$

$$\text{Area} = \frac{1}{2}\cdot\sqrt{5^2+3^2+2^2} = \frac{\sqrt{38}}{2}.$$

Rubric: 2pts for Area formula, 2pts for cross product.
1pts for Area value.

28. Find plane equation that through $(2,-1,3)$ and perpendicular to $\vec{v} = (1,-2,2) + t(3,-2,4)$.

Solution: Since it's $\perp$ to $\vec{v} = (1,-2,2) + t(3,-2,4)$.

then $(3,-2,4)$ is a normal vector to the plane

thus taking $(A, B, C) = (3, -2, 4)$, we have

$3(x-2) - 2(y+1) + 4(z-3) = 0$ ⊛ is the equation

Rubric:
2pts for equation
2pts for normal vector,
1pts for ⊛.

Section 1-4.

4 (a) $(r, \theta, z) \mapsto (r, \theta, -z)$.

Reflection with respect to xOy-plane.

(b) $(r, \theta, z) \longmapsto (r, \theta+\pi, -z)$.

Rotation about z-axis for π (clockwise or counter-clockwise) and reflection with respect to xOy-plane.

(c) $(r, \theta, z) \mapsto (-r, \theta-\frac{\pi}{4}, z) = (r, \theta+\frac{3}{4}\pi, z)$ or $= (r, \theta-\frac{5}{4}\pi, z)$.

Rotation about z axis for $\frac{5\pi}{4}$ clockwise. or $\frac{3}{4}\pi$ counter-clockwise

Rubric: 1 pts for (a), 2pts for (b), (c) each.

8 (a). In cylindrical coordinate system.

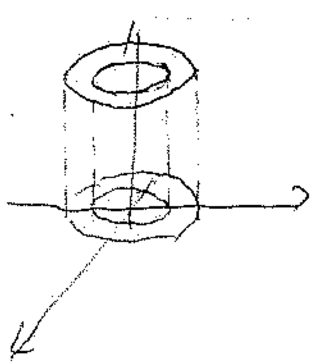
r = constant : An infinite tube with radius r about z-axis, given by : $x^2+y^2=r^2$, r fixed

θ = constant : A plane passing through the origin. given by : $x\sin\theta - y\cos\theta = 0$, θ fixed.

z = constant : A plane parallel to xOy-plane. given by : $z = z_0$, z_0 fixed.

Rubric : 3 pts for describing the right shape, 2pts for the correct position description.

10 (a).



cylindrical coordinate system.

$\{(r, \theta, z) : 1 \le r \le \frac{3}{2}, 0 \le z \le 8\}$.

Rubric : 2 pts for choosing coordinate system, 2 pts for using suitable inequality, 1 pts for final answer