Section 1-4.

2. Find the spherical coordinates of $(x, y, z) = (\sqrt{6}, -\sqrt{2}, -2\sqrt{2})$

Solution: $\rho = \sqrt{x^2+y^2+z^2} = \sqrt{6+2+8} = 4.$

So since $z = \rho\cos\phi = -2\sqrt{2}$, $\cos\phi = -\frac{\sqrt{2}}{2}$

So $\phi = \frac{3}{4}\pi$, $\sin\phi = \frac{\sqrt{2}}{2}$.

$x = \rho\sin\phi\cos\theta = \sqrt{6}$, $\cos\theta = \frac{\sqrt{3}}{2}$

So $\theta = \frac{\pi}{6}$, or $2\pi - \frac{\pi}{6}$.

While $y = \rho\sin\phi\sin\theta < 0$

So $(\rho, \theta, \phi) = (4, \frac{11\pi}{6}, \frac{3}{4}\pi)$.

Rubric: 2 pts for using correct formulas

1 pt for each coordinate, ρ, θ and ϕ.

11. S is the sphere of radius R at the origin

find equation of S in cylindrical coordinates.

Solution: In Cartesian coordinates

$S: x^2+y^2+z^2 = R^2$.

Since in (r, θ, z), $x^2+y^2 = r^2$

So $S: r^2+z^2 = R^2$ in cylindrical coordinates.

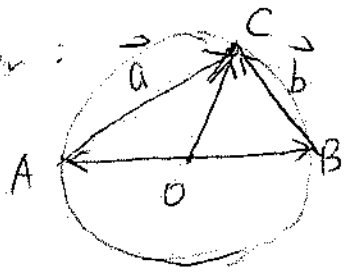
Rubric: 2pts for equation of S in some coordinates

2pts for changing coordinates, 1pts for answer.

Section 1.5.

8. Suppose T is a triangle formed by placing three points on a circle, two of which lie on the diameter. Show T is a right triangle.

Solution:



Clearly,

$$\vec{a} = \vec{OC} - \vec{OA} = \vec{OC} + \vec{OB}.$$

while $\vec{b} = \vec{OC} - \vec{OB}$.

then let $\vec{OC} = \vec{v}$, $\vec{OB} = \vec{w}$ as in #7.

we have $\vec{a} \perp \vec{b}$. □.

Rubric: 3 pts for writing $\vec{a}$, $\vec{b}$ as $\vec{v}+\vec{w}$ and $\vec{v}-\vec{w}$

2 pts for using #7 to draw conclusion.

24. Find two 2x2 matrices, A and B, s.t $AB = 0$ but $BA \neq 0$.

Solution: for example: $A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

then $AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, $BA = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

Rubric: 5 pts for a correct example of A and B.

Solution:

2. (a) $f(u,v,w) = (u^2v, we^4, 5v)$ vector-valued.

(b) $g(x) = \log\sqrt{x}$ scalar-valued.

(c) $h(x,y) = x^5y^{-3}$ scalar-valued.

Rubrics: Take off 1 pts for each wrong answer. (5 pts total)

10. Describe the behavior of the level curve $f(x,y) = c$.

Solution: (a) $f(x,y) = x^2+y^2+1 = c$.

$$x^2+y^2 = c-1$$

for $c > 1$, level curve is a circle, with radius $\sqrt{c-1}$.

for $c = 1$, level curve is a pt.

for $c < 1$, level curve is empty.

(b) $f(x,y) = 1 - x^2 - y^2 = c$.

$$x^2+y^2 = 1-c.$$

Similar to (a).

for $c < 1$, level curve is a circle with radius $\sqrt{1-c}$

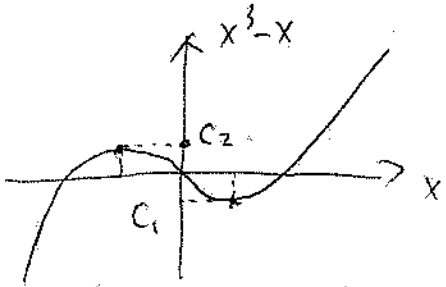
for $c = 1$, a point.

for $c > 1$, empty.

Rubric: 2pts for (a), 1 pts for (b), 2 pts for (c).

(c) $f(x,y) = x^3 - x = c$.

$$x^3 - x = c.$$



for c between c_1 and c_2. the level cure is 3 straight lines, and they merge into two lines when c goes to c_1 or c_2, and only one line left is $c > c_2$ or $c < c_1$.