Section 2.2.

8 (a) $\lim_{(x,y)\to(0,0)} \dfrac{(x+y)^2-(x-y)^2}{xy}$

$= \lim_{(x,y)\to(0,0)} \dfrac{x^2+2xy+y^2-x^2+2xy-y^2}{xy} = \lim_{(x,y)\to(0,0)} 4 = 4$

(b) $\lim_{(x,y)\to(0,0)} \dfrac{\sin xy}{y} = \lim_{(x,y)\to(0,0)} x \cdot \dfrac{\sin xy}{xy} = \lim_{(x,y)\to(0,0)} x = 0.$

(c) $\lim_{(x,y)\to(0,0)} \dfrac{x^3-y^3}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \dfrac{(x-y)(x^2+xy+y^2)}{x^2+y^2}$

$= \lim_{(x,y)\to(0,0)} (x-y)\cdot\left(1+\dfrac{xy}{x^2+y^2}\right) = 0.$

Since $|2xy| \le (x^2+y^2)$, $\left|\dfrac{xy}{x^2+y^2}\right| \le \dfrac{1}{2}$

Rubric: 1pt for (a) 2pts each for (b) and (c).

12. (a) $\lim_{x\to 0} \dfrac{\sin 2x - 2x}{x^3} \overset{L'Hopital}{=} \lim_{x\to 0} \dfrac{2\cos 2x - 2}{3x^2} \overset{L'Hopital}{=} \lim_{x\to 0} \dfrac{-4\sin 2x}{6x}$

$= -\dfrac{4}{3}$

Rubric:
1 pt for (a)
2 pts each for (b), (c).

(b) $\lim_{(x,y)\to(0,0)} \dfrac{\sin 2x - 2x + y}{x^3+y}$ Suppose limit exists

① If $y\equiv 0$, let $x\to 0$, by (a) the lim $= -\frac{4}{3}$.

② If $x\equiv 0$, let $y\to 0$, $\lim_{y\to 0} \frac{y}{y} = 1 \ne -\frac{4}{3}$,

Thus, Limit DNE.

(c) $\lim_{(x,y,z)\to(0,0,0)} \dfrac{2x^2y\cos z}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \dfrac{2x^2y}{x^2+y^2} = \lim_{(x,y)\to(0,0)} x\cdot\dfrac{2xy}{x^2+y^2}.$

$= 0$ Since $0 < \dfrac{|2xy|}{x^2+y^2} \le 1$ if $x, y \ne 0$.

14. $f(x,y,z) = \dfrac{1}{x^2+y^2+z^2-1}$.

Solution: The set where f fails to be continuous is exact where f is undefined, i.e. $x^2+y^2+z^2-1=0$

Thus is the set $\{(x,y,z) \mid x^2+y^2+z^2=1\}$

Which is the unit sphere centered at $(0,0,0)$.

Rubric: 3pts for finding the condition for f to be not continuous, 2pts for description of the set.

Section 2.3.

1. (a) $f(x,y) = x \cdot y$, $\dfrac{\partial f}{\partial x} = y$, $\dfrac{\partial f}{\partial y} = x$.

(b) $f(x,y) = e^{xy}$, $\dfrac{\partial f}{\partial x} = ye^{xy}$, $\dfrac{\partial f}{\partial y} = xe^{xy}$.

(c) $f(x,y) = x\cos x\cos y$, $\dfrac{\partial f}{\partial x} = \cos x\cos y - x\sin x\cos y$,

$\dfrac{\partial f}{\partial y} = -x\cos x\sin y$.

(d). $f(x,y) = (x^2+y^2)\log(x^2+y^2)$

$$\frac{\partial f}{\partial x} = 2x\left(1+\log(x^2+y^2)\right)$$

$$\frac{\partial f}{\partial y} = 2y\left(1+\log(x^2+y^2)\right).$$

Rubric: 1 pt each. (5 pts total still).

10. (a) $f(x,y) = (e^x, \sin xy)$.

$$\vec{D} f(x,y) = \begin{bmatrix} e^x & 0 \\ y\cos xy & x\cos xy \end{bmatrix}$$

(b) $f(x,y,z) = (x-y, y+z)$.

$$\vec{D} f(x,y,z) = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

(c) $f(x,y) = (x+y, x-y, xy)$.

$$\vec{D} f(x,y) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ y & x \end{bmatrix}$$

Rubric: 1 pt each.

(d). $f(x,y,z) = (x+y, y-5z, x-y)$

$$\vec{D} f(x,y,z) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -5 \\ 1 & -1 & 0 \end{bmatrix}$$

18. (a) $\nabla f(x,y) = (e^{y^2} - 2xye^{x^2}, 2xye^{y^2} - e^{x^2})$, $f(1,2) = e^4 - 2e$

$\nabla f(x,y)\big|_{(1,2)} = (e^4 - 4e, 4e^4 - e)$

Tangent plane: $z = e^4 - 2e + (e^4 - 4e)(x-1) + (4e^4 - e)(y-2)$.

(b) $g(x,y) = x^2 - y^2$, $\nabla g(x,y) = (2x, -2y)$.

The planes are parallel if $\nabla f = \nabla g$, so $\begin{cases} 2x = e^4 - 4e, \\ -2y = 4e^4 - e. \end{cases}$

So $(x,y) = (\frac{e^4}{2} - 2e, \frac{e}{2} - 2e^4)$.

Rubric: 3 pts for (a) 2pts for (b).