Section 2.4.

8. $\vec{c}(t) = (\sin 3t)\vec{i} + (\cos 3t)\vec{j} + 2t^{\frac{3}{2}}\vec{k}$

Solution: velocity vector $\vec{c}\,'(t) = 3\cos 3t\,\vec{i} - 3\sin 3t\,\vec{j} + 3t^{\frac{1}{2}}\vec{k}$.

Rubric: 2pts for correct definition of velocity vector
1 pt each term.

18. $(\cos^2 t, 3t - t^3, t)$; $t=0$, Find the tangent line.

Solution: $\vec{l}(t) = \vec{c}(t_0) + (t - t_0)\vec{c}\,'(t_0)$.

$\vec{c}\,'(t) = (-2\cos t \sin t, 3 - 3t^2, 1)$

$\vec{c}(0) = (1, 0, 0) \quad \vec{c}\,'(0) = (0, 3, 1)$.

So $\vec{l}(t) = (1, 0, 0) + (t-0)\cdot(0, 3, 1)$
$= (1, 3t, t)$.

Rubric: 2pts for formula of tangent line,
3pts for the answer.

Section 2.5.

8. $f(u, v, w) = (e^{u-w}, \cos(u+v) + \sin(u+v+w))$
$g(x, y) = (e^x, \cos(y-x), e^{-y})$

Solution: $f \circ g = f(e^x, \cos(y-x), e^{-y})$
$= \left(e^{e^x - e^{-y}}, \cos(e^x + \cos(y-x)) + \sin(e^x + \cos(y-x) + e^{-y})\right)$

$\vec{D}(f \circ g)(0,0) = \vec{D} f(g(0,0)) \cdot \vec{D} g(0,0)$

$$= \begin{pmatrix} 1 & 0 & -1 \\ \cos 3 - \sin 2 & \cos 3 - \sin 2 & \cos 3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \cos 3 - \sin 2 & -\cos 3 \end{pmatrix}$$

Rubric:
2 pts for $f \circ g$
3 pts for $\vec{D}(f \circ g)(0,0)$

12. $h(x,y,z) = (xyz,\ e^{xz},\ x\sin y,\ -\frac{9}{x},\ 17)$

$g(u,v) = (v^2+2u,\ \pi,\ 2\sqrt{u})$ Find $\vec{D}(h\circ g)(1,1)$.

Solution: $\vec{D}(h\circ g)(1,1) = \vec{D}h(g(1,1)) \cdot \vec{D}g(1,1)$

$$g(1,1) = (3,\ \pi,\ 2).$$

$$\vec{D}g(1,1) = \begin{pmatrix} 2 & 2 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\vec{D}h(3,\pi,2) = \begin{pmatrix} 2\pi & 6 & 3\pi \\ 2e^6 & 0 & 3e^6 \\ 0 & -3 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

So $$\vec{D}(h\circ g)(1,1) = \begin{pmatrix} 7\pi & 4\pi \\ 7e^6 & 4e^6 \\ 0 & 0 \\ 2 & 2 \\ 0 & 0 \end{pmatrix}$$

Rubric: 2pts for chain rule, 1pt for each derivative matrix.

Section 2.6.

6. Find a vector normal to $x^3+xy+y^3=11$ at $(1,2)$.

Solution: let $f(x,y) = x^3+xy+y^3$.

$$\nabla f\big|_{(1,2)} = (3x^2+y,\ x+3y^2)\big|_{(1,2)}$$
$$= (5,\ 13)$$

So $(5,13)$ is normal to the curve at $(1,2)$.

Rubric: 3 pts for defining f and finding ∇f

2 pts for answer.

14. Verify THM 13, 14 for $f(x,y,z) = x^2+y^2+z^2$.

Solution: f is the distance square to $(0,0,0)$.

Clearly f increases the fastest at point (x,y,z) along the direction away from $(0,0,0)$ which is the same as (x,y,z).

While THM 13 says this is $\nabla f|_{(x,y,z)}$

$$= (2x, 2y, 2z) \checkmark.$$

For THM 14, $f(x,y,z) = k$ is a sphere with radius $\sqrt{k}$, Clearly $\nabla f(x_0,y_0,z_0)$ $= (2x_0, 2y_0, 2z_0)$ is normal to the sphere at (x_0, y_0, z_0). So THM 14 $\checkmark$.

Rubric: 3pts for THM 13, 2pts for THM 14.