Section 3.4.

2. Rectangles with perimeter P, find the max. area.

Solution: Area $= f(x,y) = x\cdot y$.

with restriction $g(x,y) = 2(x+y) = P$.

$$\nabla f = (y, x), \quad \nabla g = (2,2).$$

$$\nabla f = \lambda \nabla g \Rightarrow \begin{cases} y = 2\lambda \\ x = 2\lambda. \end{cases}$$

with $2(x+y) = P \Rightarrow x = y = \frac{P}{4}$.

So Max. Area $= \frac{P^2}{16}$ with $x = y = \frac{P}{4}$.

Rubric: 3 pts for setting up L-multipliers

2 pts for correct answer.

4. $f(x,y) = x - y$, subject to $x^2 - y^2 = 2$

$g(x,y) = x^2 - y^2$

Solution: $\nabla f = (1, -1)$, $\nabla g = (2x, -2y)$

So $\nabla f = \lambda \nabla g \Rightarrow \begin{cases} 1 = 2\lambda x \\ -1 = -2\lambda y. \end{cases} \xRightarrow{\text{Clearly, } \lambda \neq 0} \begin{cases} x = \frac{1}{2\lambda} \\ y = \frac{1}{2\lambda} \end{cases}$

with $x^2 - y^2 = 2$.

so $x = y \Rightarrow x^2 - y^2 = 0$, No solution

So No extrema.

Rubric: 3pts for setting up L-multipliers

2pts for conclusion.

36. Maximize $Q(x,y) = xy$, subject to $C(x,y) = 2x+3y=10$.

Solution: $\nabla Q = (y, x)$

$\nabla C = (2,3)$

So $\nabla Q = \lambda \nabla C \Rightarrow \begin{cases} y = 2\lambda \\ x = 3\lambda. \end{cases}$

with $2x+3y=10 = 6\lambda + 6\lambda = 12\lambda$

$\Rightarrow \lambda = \frac{5}{6}$.

So Q has max. $= 2\lambda\cdot 3\lambda = 6\cdot\lambda^2 = \frac{25}{6}$.

with $\begin{cases} x = \frac{5}{2} \\ y = \frac{5}{3}. \end{cases}$

Rubric: 3pts for setting up L-multiplier

2pts for correct answer.

Section 4.1.

3. $\vec{r}(t) = \sqrt{2}t\,\vec{i} + e^{t}\vec{j} + e^{-t}\vec{k}$, at $t=0$.

$\vec{r}'(t) = \sqrt{2}\,\vec{i} + e^{t}\vec{j} - e^{-t}\vec{k}$, $\vec{r}''(t) = e^{t}\vec{j} + e^{-t}\vec{k}$.

$\vec{r}'(0) = (\sqrt{2}, 1, -1)$, $\vec{r}''(0) = \vec{j} + \vec{k} = (0,1,1)$

$\vec{\ell}(t) = \sqrt{2}t\,\vec{i} + (1+t)\vec{j} + (1-t)\vec{k}$.

Rubric: 1pt for each formula above.

9. $\vec{c}(t) = (a\cos t, a\sin t, bt)$

Solution: $\vec{v}(t) = (-a\sin t, a\cos t, b)$.

$\vec{a}(t) = (-a\cos t, -a\sin t, 0)$.

which is parallel to the xy-plane.

Rubric: 3 pts for computing $\vec{a}$

2 pts for conclusion.

20.

Proof: $f(t) = \|\vec{r}(t)\|^2 = \vec{r}(t)\cdot\vec{r}(t)$

at local max/min of f

$f'(t) = 0$

While $f'(t) = \vec{r}'(t)\cdot\vec{r}(t) + \vec{r}(t)\cdot\vec{r}'(t)$

$= 2\cdot\vec{r}'(t)\cdot\vec{r}(t) = 0$

implies $\vec{r}'(t) \perp \vec{r}(t)$.

Rubric: 3 pts for computing the derivative

of $\|\vec{r}(t)\|$ or $\|\vec{r}(t)\|^2$.

2 pts for drawing conclusion.