Section 5.1

6. Solution: Cross Section Area

$= $ Area of a 3×5 rectangle

$= 3 \times 5 = 15.$

Volume $= 15 \times$ Height $= 15 \times 7$

$= 105$

Rubric: 3 pts for correct formula, 2pts for answer.

Section 5.2.

2(c). $\iint_R \sin(x+y)\,dx\,dy. \quad R = [0,1]\times[0,1].$

$$= \int_0^1 \int_0^1 \sin(x+y)\,dx\,dy$$

$$= \int_0^1 -\cos(x+y)\Big]_{x=0}^{1} dy$$

$$= \int_0^1 -\cos(y+1) + \cos y \, dy.$$

$$= \Big[-\sin(y+1) + \sin y\Big]_{y=0}^{1} = -\sin 2 + \sin 1.$$

Rubric: 2 pts for each integral

1 pt for answer.

2(d) $\iint_R (x^2+2xy+y\sqrt{x})\,dx\,dy$

$$= \int_0^1\int_0^1 (x^2+2xy+yx^{\frac{1}{2}})\,dx\,dy.$$

$$= \int_0^1 \left(\frac{x^3}{3}+x^2y+\frac{2}{3}yx^{\frac{3}{2}}\right)\Big]_{x=0}^{1} dy$$

$$= \int_0^1 \left(\frac{1}{3}+y+\frac{2}{3}y\right) dy.$$

$$= \left(\frac{1}{3}y+\frac{y^2}{2}+\frac{1}{3}y^2\right)\Big]_0^1 = \frac{1}{3}+\frac{1}{2}+\frac{1}{3}=\frac{7}{6}$$

Rubric: 2 pts for each integral, 1 pt for answer.

Section 5.3.

12. $\iint_D \cos y\,dx\,dy$ $D=$ bounded by $y=x$, $y=2x$, $x=\pi$, $x=2\pi$.

$$= \int_\pi^{2\pi}\int_x^{2x} \cos y\,dy\,dx$$

$$= \int_\pi^{2\pi} (\sin 2x-\sin x)\,dx.$$

$$= \left(-\frac{\cos 2x}{2}+\cos x\right)\Big]_\pi^{2\pi} = -\frac{1}{2}+1-\left(-\frac{1}{2}-1\right)=2$$

Rubric: 3 points for use the right order of integration, 2pts for answer.

Section 5.4.

2. $\int_0^1 \int_y^1 \sin(x^2)\,dx\,dy.$

$$\begin{cases} y \le x \le 1 \\ 0 \le y \le 1 \end{cases} \iff \begin{cases} 0 \le y \le x \\ 0 \le x \le 1. \end{cases}$$

$$= \int_0^1 \int_0^x \sin(x^2)\,dy\,dx$$

$$= \int_0^1 x \sin(x^2)\,dx = \frac{1}{2} \int_0^1 \sin x^2 \, dx^2$$

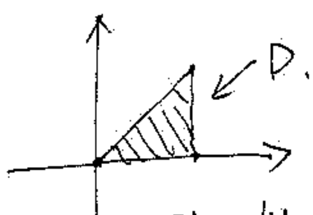
$$= \frac{1}{2} \left(-\cos x^2\right)\Big|_0^1 = -\frac{1}{2}\cos 1 + \frac{1}{2}.$$

Rubric: 3pts for ~~the~~ changing the order

2pts for answer.

10. Show $\frac{1}{6} \le \iint_D \frac{dA}{y-x+3} \le \frac{1}{4}$

Proof:



$\text{Area}(D) = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

Clearly, in D, $0 \le y \le x$

so $-1 \le -x \le y - x \le 0$

thus $2 \le y - x + 3 \le 3$, $\frac{1}{3} \le \frac{1}{y-x+3} \le \frac{1}{2}$.

By mean value inequality

$$\frac{1}{6} = \frac{1}{3} \times \text{Area}(D) \le \iint_D \frac{dA}{y-x+3} \le \frac{1}{2} \times \text{Area}(D) = \frac{1}{4} \quad \square.$$

Rubric: 3 pts for finding min/max of integrand.

2pts for finishing the proof.