Section 5.5.

4. $\iiint_B e^{-xy} y \, dx\, dy\, dz$, $B = [0,1] \times [0,1] \times [0,1]$.

$$= \int_0^1 \int_0^1 \int_0^1 e^{-xy} y \, dx\, dy\, dz.$$

$$= -\int_0^1 e^{-xy} \Big]_{x=0}^{1} dy = -\int_0^1 (e^{-y} - 1)\, dy = -\left(-e^{-y} - y\right)\Big|_0^1$$

$$= -\left(-e^{-1} - 1\right) + (-1) = e^{-1}.$$

Rubric: 2pts for answer,
1 pt for each integral.

16. $\int_0^1 \int_0^x \int_0^y (y + xz)\, dz\, dy\, dx$

$$= \int_0^1 \int_0^x \left(y + \frac{xy^2}{2}\right) dy\, dx.$$

$$= \int_0^1 \left(\frac{x^2}{2} + \frac{x^4}{6}\right) dx = \frac{1}{6} + \frac{1}{24} = \frac{5}{24}.$$

Rubric: 1 pt for each integral, 2 pts for answer.

Section 6.1.

2. (a) one-to-one and onto

Rubric: 1 pt each.

(b) not one-to-one, since $T(x,y,z) = T(-x,-y,z)$.
not onto, since $|xy| \geq |y \cdot \sin x|$.

(c) not one-to-one, since $T(1,1,1) = T(-1,-1,1)$
not onto, since $(-1,-1,-1)$ has no pre-image

(d) One-to-one, since $f(x) = e^x$ is one-to-one
not clearly not onto

14. (a) $\int_0^1 \int_0^{x^2} xy \, dy \, dx$.

Let $u = x^2$, $v = y$, $T(x,y) = (x^2, y)$.

then $du = 2x \, dx$. $T(u,v) = (\sqrt{u}, v)$.

$= \int_0^1 \int_0^u \frac{1}{2} v \, dv \, du = \frac{1}{2} \iint_{D^*} v \, dv \, du$.

(b) $\frac{1}{2} \iint_{D^*} u \, du \, dv$

$= \frac{1}{2} \int_0^1 \int_0^u v \, dv \, dy$

$= \frac{1}{2} \int_0^1 \frac{u^2}{2} \, du = \frac{1}{2} \frac{u^3}{6} \Big]_0^1 = \frac{1}{12}$.

Rubric: 3 pts for (a) 2 pts for (b).

7. $-u^2+4u$ and v are monotone over $[0,1]$ as function of u / v respectively

So $D = [-0^2+0, -1^2+4\times1] \times [0,1]$

$= [0,3] \times [0,1]$

and T is one-to-one.

Rubric: 3 pts for D, 2pts for T.

Section 6.2.

2 (a) $\iint_R (5x+y)^3 (x+9y)^4 \, dA$.

$$\begin{cases} u = 5x+y \\ v = x+9y \end{cases}$$

$$|J| = \begin{vmatrix} 5 & 1 \\ 1 & 9 \end{vmatrix} = 44.$$

(b) $\iint_{R^2} \sin(6x+7y) \cdot (x-3y) \, dA$.

$$\begin{cases} u = 6x+7y \\ v = x-3y \end{cases}$$

$$|J| = \begin{vmatrix} 6 & 7 \\ 1 & -3 \end{vmatrix} = 25.$$

Rubric:
2 pts for (a)
3 pts for (b)