Section 6.2 22.

Solution: $\int_0^\infty e^{-4x^2} dx.$

$= \lim_{a\to\infty} \int_0^a e^{-4x^2} dx$

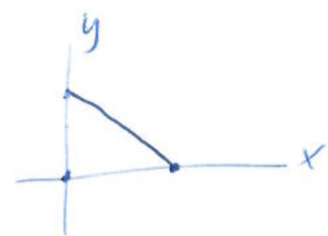
$= \lim_{a\to\infty} \int_0^{2a} e^{-y^2} \frac{dy}{2}$ ↙ since e^{-x^2} is an even function

$= \frac{1}{2} \int_0^\infty e^{-y^2} dy = \frac{1}{4} \int_{-\infty}^{\infty} e^{-y^2} dy = \frac{\sqrt{\pi}}{4}.$

Rubric: 3 pts for ~~tor~~ transform into the form $\int_{-\infty}^{\infty} e^{-x^2} dx.$ 2pts for the answer.

Section 6.3.

4. Average of $f(x,y) = e^{x+y}$ over triangle $(0,0)$, $(0,1)$, $(1,0)$



Solution:

Average of f over Δ

$= \dfrac{\iint_\Delta f(x,y)\,dx\,dy}{\text{Area}(\Delta)}$

$= \dfrac{\int_0^1 \int_0^{1-y} e^{x+y}\,dx\,dy}{\frac{1}{2}}$

$= 2\int_0^1 e^{x+y}\Big]_{x=0}^{1-y} dy = 2\int_0^1 e - e^y\,dy$

$= 2e - 2e + 2 = 2.$

Rubric: 2 pts for correct formulation

3pts for answer.

16. Find the average of e^{-z} over $x^2+y^2+z^2 \le 1$

Solution:

$$\text{Average} = \frac{\iiint_{x^2+y^2+z^2\le 1} e^{-z}\,dx\,dy\,dz}{\text{Vol of unit ball}}$$

In cylindrical coordinates

$$= \frac{\int_0^{2\pi}\int_0^1\int_0^{\sqrt{1-z^2}} r\cdot e^{-z}\,dr\,dz\,d\theta}{\frac{4}{3}\pi}$$

$$= 3\cdot\frac{2\pi}{4\pi}\int_0^1 \frac{1-z^2}{2}e^{-z}\,dz$$

$$= \frac{3}{4}\int_0^1 (1-z^2)e^{-z}\,dz = \frac{3}{4}\left(\frac{4}{e}-1\right)$$

Rubric: 3 pts for correct formulation + Change of variables
2 pts for answer.

Section 7.1. 4. $\frac{(x-2)^2}{4}+\frac{(y-3)^2}{9}=1$

Solution: $\left(\frac{x-2}{2}\right)^2+\left(\frac{y-3}{3}\right)^2=1$ (*)

Let $\frac{x-2}{2}=\cos\theta$, $\frac{y-3}{3}=\sin\theta$.

we have $\begin{cases} x = 2\cos\theta+2 \\ y = 3\sin\theta+3 \end{cases}$ $\theta\in[0,2\pi)$

Rubric: 3 pts for finding the form (*)

2 pts for answer.

10 (a) $\int_{\vec{C}} (x+y+z)\,ds \quad \vec{C}: t \to (\sin t, \cos t, t),$

$t \in [0, 2\pi]$

Solution:

$$= \int_0^{2\pi} (\sin t + \cos t + t)\sqrt{\cos^2 t + \sin^2 t + 1}\; dt$$

$$= \sqrt{2} \int_0^{2\pi} \sin t + \cos t + t \; dt.$$

$$= \sqrt{2} \; \frac{t^2}{2}\Big]_0^{2\pi} = 2\sqrt{2}\pi^2.$$

Rubric: 3 pts for correct formula 2 pts for answer.

24. $\int_{y=e^x, 0\le x\le 1} y^2 \, ds$

Solution: we parametrize $y = e^x$ by (t, e^t), $0 \le t \le 1$

then $\int_{\vec{C}} y^2 \, ds$

$$= \int_0^1 e^{2t}\sqrt{1+e^{2t}}\, dt$$

$$= \frac{1}{3}\left((1+e^2)^{\frac{3}{2}} - 2\sqrt{2} \right)$$

Rubric: 2 pts for parametrizing the curve

1 pt for the formulation of integral

2 pts for final answer.