Section 7.2. 4. 8. 11. 14. 17. 18.

4 (a). $\int_{\vec{c}} x\,dy - y\,dx$. $\quad \vec{c}(t) = (\cos t, \sin t)$

$0 \le t \le 2\pi$, $x = \cos t$, $y = \sin t$

$dy = \cos t\,dt$, $dx = -\sin t\,dt$

$$= \int_0^{2\pi} (\cos^2 t + \sin^2 t)\,dt = 2\pi.$$

(b). $\int_{\vec{c}} x\,dx + y\,dy$ $\quad \vec{c}(t) = (\cos \pi t, \sin \pi t)$

$0 \le t \le 2$, $x = \cos \pi t$, $y = \sin \pi t$

$dx = -\pi \sin \pi t\,dt$, $dy = \pi \cos \pi t\,dt$

$$= \int_0^2 -\pi \sin \pi t \cdot \cos \pi t\,dt + \pi \sin \pi t \cos \pi t\,dt = 0.$$

(c). $\int_{\vec{c}} yz\,dx + xz\,dy + xy\,dz$ $\quad \vec{c} = (1,0,0)$ to $(0,1,0)$ to $(0,0,1)$.

$$= \int_{\vec{c}} \nabla(xyz) \cdot d\vec{s}$$

$$= (x \cdot y \cdot z)\Big]_{(0,0,1)} - (x \cdot y \cdot z)\Big]_{(1,0,0)} = 0.$$

(d) $\int_{\vec{c}} x^2\,dx - xz\,dy + dz$. $\quad \vec{c}: z = x^2, y = 0$ from $(-1,0,1)$ to $(1,0,1)$

$$= \int_{-1}^{1} x^2\,dx - 0 + 2x\,dx$$

$$= \int_{-1}^{1} x^2 + 2x\,dx = \frac{x^3}{3} + x^2\Big]_{-1}^{1} = \frac{2}{3}$$

8. $\int_{\vec{c}} \vec{F} \cdot d\vec{s} \quad \vec{F} = (y, 2x, y)$, $\vec{c}(t) = (t, t^2, t^3)$.

$0 \le t \le 1$.

$$\int_{\vec{c}} \vec{F} \cdot d\vec{s} = \int_0^1 t^2 dt + (2t)^2 dt + t^2 \cdot 3t^2 dt$$

$$= \int_0^1 5t^2 + 3t^4 dt = \frac{5}{3} + \frac{3}{5}.$$

11. $\int (x, y) \cdot d\vec{s} = \int \nabla(x \cdot y) \cdot d\vec{s} = 0$

since C is closed.

14. $\vec{F} = (z^3 + 2xy)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$

$\vec{F} = \nabla f$ if $f = xz^3 + x^2 y$.

So $\int_C \vec{F} \cdot d\vec{s} = 0$ if C is Closed.

17. $\int_C 2xyz\,dx + x^2 z\,dy + x^2 y\,dz$

$$= \int_C \nabla(x^2 yz) \cdot d\vec{s}.$$

$$= x^2 yz \Big]_{(1,2,4)} - x^2 yz \Big]_{(1,1,1)}$$

$$= 8 - 1 = 7.$$

18. $\nabla f(x,y,z) = (2xyz e^{x^2},\ z e^{x^2},\ y e^{x^2})$

So $f = yze^{x^2} + C$

since $f(0,0,0)=5$, $C=5$

$f(1,1,2) = 2e^1 + 5 = 2e+5$.

Section 7.3. 1, 2, 4, 6, 7.

1. $(x, y, z) = (2u,\ u^2+v,\ v^2)$

$\vec{T}_u = (2,\ 2u,\ 0)\Big|_{(0,1,1)} = (2,0,0)$

$\vec{T}_v = (0,\ 1,\ 2v)_{(0,1,1)} = (0,1,2)$.

$\vec{n} = \vec{T}_u \times \vec{T}_v = (0, -4, 2)$.

So tangent plane:

$0\cdot(x-0) + (-4)(y-1) + 2(z-1) = 0$.

$\Rightarrow z-1-4y+4=0$

2. $(x, y, z) = (u^2 - v^2,\ u+v,\ u^2+4v)$

$$\vec{T_u} = (2u,\ 1,\ 2u)\Big|_{(-\frac{1}{4},\frac{1}{2},2)} = (0, 1, 0)$$

$$\vec{T_v} = (-2v,\ 1,\ 4)\Big|_{(-\frac{1}{4},\frac{1}{2},2)} = (-2, 1, 4).$$

$$\vec{n} = \vec{T_u} \times \vec{T_v} = (4, 0, 2)$$

Tangent plane:

$$4(x+\tfrac{1}{4}) + 2(z-2) = 0.$$

4. For 1. since $(2, 2u, 0)$, $(0, 1, 2v)$ never are parallel.

It's always regular.

For 2. Non-regular if $(2u, 1, 2u) \times (-2v, 1, 4) = \vec{0}$

$$\Rightarrow (4-2u,\ -8u-4uv,\ 2u+2v) = 0$$

$$\Rightarrow \begin{cases} u = 2 \\ v = -2. \end{cases}$$

So it's regular if $(u, v) \neq (2, -2)$.

6. $\vec{\Phi}(u,v) = (u-v,\ u+v,\ 2uv)$

$$\vec{T_u} = (1, 1, 2v) \qquad \vec{T_u} \times \vec{T_v} = (2u-2v,\ -2u-2v,\ 2)$$

$$\vec{T_v} = (-1, 1, 2u) \qquad \neq \vec{0}$$

So $\vec{\Phi}$ is always regular.

7. (a) (iii) (b) (i) (c) (ii) (d) (iv).

Section 7.4 4, 6, 24

4. $(x, y, z) = ((R+\cos\phi)\cos\theta, (R+\cos\phi)\sin\theta, \sin\phi)$.

$$A(S) = \iint_D \sqrt{\left(\frac{\partial(x,y)}{\partial(\theta,\phi)}\right)^2 + \left(\frac{\partial(y,z)}{\partial(\theta,\phi)}\right)^2 + \left(\frac{\partial(x,z)}{\partial(\theta,\phi)}\right)^2}\, d\theta\, d\phi.$$

$$\begin{pmatrix} -(R+\cos\phi)\sin\theta & -\sin\phi\cos\theta \\ (R+\cos\phi)\cos\theta & -\sin\phi\sin\theta \\ 0 & \cos\phi \end{pmatrix}$$

So $\left(\frac{\partial(x,y)}{\partial(\theta,\phi)}, \frac{\partial(y,z)}{\partial(\theta,\phi)}, \frac{\partial(x,z)}{\partial(\theta,\phi)}\right) = \Big(\sin^2\theta\sin\phi R + \sin^2\theta\sin\phi\cos\phi + \cos^2\theta\sin\phi R + \cos^2\theta\sin\phi\cos\phi,$

$\cos^2\phi\cos\theta + R\cos\phi\cos\theta,$

$-(R\cos\phi + \cos^2\phi)\sin\theta\Big)$

$= (\sin\phi R + \sin\phi\cos\phi, \cos\phi\cos\theta(R+\cos\phi),$

$-(R+\cos\phi)\cos\phi\sin\theta)$.

$$A(S) = \int_0^{2\pi}\int_0^{2\pi} \sqrt{(\sin^2\phi + \cos^2\phi\cos^2\theta + \cos^2\phi\sin^2\theta)(R+\cos\phi)^2}\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{2\pi} R+\cos\phi\, d\phi\, d\theta = (2\pi)^2 \cdot R.$$

6. $z = xy$, $x^2+y^2 \le 2$.

$(x, y, z) = (r\cos\theta, r\sin\theta, r^2\cos\theta\sin\theta)$. $D = [0,\sqrt{2}] \times [0, 2\pi]$.

$$\begin{array}{c|ccc} & x & y & z \\ \hline r & \cos\theta & \sin\theta & 2r\cos\theta\sin\theta \\ \theta & -r\sin\theta & r\cos\theta & r^2(-\sin^2\theta+\cos^2\theta) \end{array}$$

$\frac{\partial(x,y)}{\partial(r,\theta)} = r\cos^2\theta + r\sin^2\theta = r$

$\frac{\partial(y,z)}{\partial(r,\theta)} = r^2(\cos^2\theta\sin\theta - \sin^3\theta - 2\cos^2\theta\sin\theta)$

$= r^2 \sin\theta(-\sin^2\theta + \cos^2\theta = r^2\sin\theta$

$\frac{\partial(x,z)}{\partial(r,\theta)} = r^2(-\sin^2\theta\cos\theta + \cos^3\theta + 2\cos\theta\sin^2\theta)$

$= r^2\cos\theta$.

$$A(S) = \int_0^{2\pi}\int_0^{\sqrt{2}} \sqrt{r^2 + r^4\sin^2\theta + r^4\cos^2\theta}\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\sqrt{2}} r\sqrt{1+r^2}\, dr\, d\theta$$

$$= 2\pi\cdot\left(\sqrt{3} - \frac{1}{3}\right).$$

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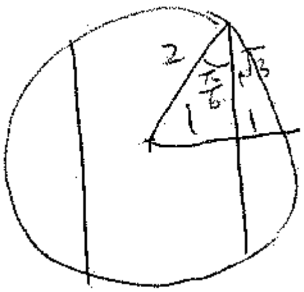

$$A = 2\pi\cdot 2\int_0^{\sqrt{3}} |x|\sqrt{1+(f'(x))^2}\, dx.$$

$$f(x) = \sqrt{4-x^2},\quad f'(x) = \frac{-x}{\sqrt{4-x^2}}$$

$$A = 4\pi\int_0^{\sqrt{3}} x\cdot\sqrt{1+\frac{x^2}{4-x^2}}\, dx$$

$$= 4\pi\int_0^{\sqrt{3}} \frac{2x}{\sqrt{4-x^2}}\, dx = 8\pi.$$

Section 7.5 2, 4, 6, 16, 20.

2. $f(x,y,z) = z+6$

$$\Phi(u,\theta) = \left(u, \frac{\theta}{3}, \theta\right)$$

$$\vec{T}_u = (1, 0, 1) \qquad \vec{T}_\theta = \left(0, \frac{1}{3}, 0\right).$$

$$\|\vec{T}_u \times \vec{T}_\theta\| = \sqrt{\frac{1}{9} + 0 + \frac{1}{9}} = \frac{\sqrt{2}}{3}$$

$$\iint f\, dS = \int_0^2\int_0^3 (\theta+6)\cdot\frac{\sqrt{2}}{3}\, d\theta\, du.$$

$$= \frac{2}{3}\sqrt{2}\left(\frac{\theta^2}{2} + 6\theta\right)\Big]_0^3 = \frac{2}{3}\sqrt{2}\left(\frac{9}{2} + 18\right).$$

4 $\iint_S (x+z)\,dS$.

$\Phi(x,\theta) = (x,\ 2\cos\theta,\ 2\sin\theta)$.

$\vec{T_x} = (1,\ 0,\ 0)$

$\vec{T_\theta} = (0,\ -2\sin\theta\ 2\cos\theta)$

$\vec{T_x}\times\vec{T_\theta} = (\cancel{2\cos\theta},\ 0,\ -2\sin\theta)$.

$\|\vec{T_x}\times\vec{T_\theta}\| = 2$.

$$\iint_S (x+z)\,dS = \int_0^{2\pi}\int_0^5 (x+2\sin\theta)\,dx\,d\theta$$

$$= 2\pi \times \frac{25}{2} = 25\pi.$$

6. $\iint_S (x^2z+y^2z)\,dS$.

$\Phi(r,\theta) = (r\cos\theta,\ r\sin\theta,\ 4+r(\cos\theta+\sin\theta))$.

$\vec{T_r} = (\cos\theta,\ \sin\theta,\ \cos\theta+\sin\theta)$

$\vec{T_\theta} = (-r\sin\theta,\ r\cos\theta,\ -r\sin\theta+r\cos\theta)$.

$\vec{T_r}\times\vec{T_\theta} = r\cdot(-\sin^2\theta+\sin\theta\cos\theta-\cos^2\theta-\sin\theta\cos\theta,\ \cancel{r}\cos\theta\sin\theta-\cos^2\theta-\sin\theta\cos\theta\ \cancel{r}-\sin^2\theta,$

$\cos^2\theta+\sin^2\theta)$

$= r(-1,\ -1,\ 1)$.

$\vec{T_r}\times\vec{T_\theta} = \sqrt{3}\,r$.

$$\iint_S (x^2z+y^2z)\,dS = \int_0^2\int_0^{2\pi}(4+r(\cos\theta+\sin\theta))\cdot r^3\cdot\sqrt{3}\,dr\,d\theta$$

$$= 2\pi\int_0^2 4\sqrt{3}\,r^3\,dr = 8\pi\sqrt{3}\cdot 4 = 32\sqrt{3}\pi.$$

16. $M = \int_0^{2\pi}\int_0^{\pi} R^2 \sin^2\phi \quad R^2 \sin\phi \, d\phi \, d\theta.$

$= 2\pi R^4 \int_0^{\pi} \sin^3\phi \, d\phi = \frac{8\pi}{3} R^4.$

20. $\iint_S (1-z)\, dS.$

$\Phi(r,\theta) = (r\cos\theta, r\sin\theta, 1-r^2).$

$\vec{T_r} = (\cos\theta, \sin\theta, -2r)$

$\vec{T_\theta} = (-r\sin\theta, r\cos\theta, 0).$

$\vec{T_r} \times \vec{T_\theta} = (2r^2\cos\theta, 2r^2\sin\theta, r).$

$\|\vec{T_r} \times \vec{T_\theta}\| = \sqrt{4r^4 + r^2} = r\sqrt{4r^2+1}.$

$\iint_S (1-z)\, dS = \int_0^1 \int_0^{2\pi} r^3\sqrt{4r^2+1}\, d\theta\, dr$

$= 2\pi \cdot \frac{1}{120}(1+25\sqrt{5}).$

Section 7.6. 2, 4, 9.

2. $\iint_S \vec{F} \cdot d\vec{S}. \quad \vec{F} = (x, y, z^2)$

$\Phi(u,v) = (2\sin u, 3\cos u, v).$

$\vec{T_u} = (2\cos u, -3\sin u, 0)$

$\vec{T_v} = (0, 0, 1).$

$\vec{T_u} \times \vec{T_v} = (-3\sin u, -2\cos u, 0).$

$\vec{F} \cdot (\vec{T_u} \times \vec{T_v}) = (2\sin u, 3\cos u, v^2) \cdot (-3\sin u, -2\cos u, 0)$

$= -6.$

$$\iint_S \vec{F}\cdot d\vec{S} = \int_0^{2\pi}\int_0^1 -6\, du\, du = -12\pi.$$

4. $\iint_S \vec{F}\cdot d\vec{S}$, $\vec{F} = (2x, -2y, z^2)$.

$\vec{\Phi}(\theta, z) = (2\cos\theta, 2\sin\theta, z)$.

$\vec{T_\theta} = (-2\sin\theta, 2\cos\theta, 0)$

$\vec{T_z} = (0, 0, 1)$

$\vec{T_\theta}\times\vec{T_z} = (2\cos\theta, 2\sin\theta, 0)$.

$\vec{F}\cdot \vec{T_\theta}\times\vec{T_z} = (4\cos\theta, -4\sin\theta, z^2)\cdot(2\cos\theta, 2\sin\theta, 0)$

$= 8\cos^2\theta - 8\sin^2\theta = 8\cos 2\theta$.

$\iint_S \vec{F}\cdot d\vec{S} = \int_0^1\int_0^{2\pi} 8\cos 2\theta\, d\theta\, dz = 0$.

9. $\iint_S (\nabla\times\vec{F})\cdot d\vec{S}$, $\vec{F} = (y, -x, zx^3y^2)$

$$\nabla\times\vec{F} = \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & -x & zx^3y^3 \end{pmatrix}$$

$= (3zx^3y^2, -3zx^2y^3, -2)$.

$\Phi(\theta,\phi) = (\cos\theta\sin\phi, \sin\theta\sin\phi, \frac{1}{\sqrt{3}}\cos\phi)$.

$\vec{T_\theta} = (-\sin\theta\sin\phi, \cos\theta\sin\phi, 0)$

$\vec{T_\phi} = (\cos\theta\cos\phi, \sin\theta\cos\phi, -\frac{1}{\sqrt{3}}\sin\phi)$.

$\vec{T_\theta}\times\vec{T_\phi} = (-\frac{1}{\sqrt{3}}\cos\theta\sin^2\phi, -\frac{1}{\sqrt{3}}\sin\phi\sin^2\phi, \sin\phi\cos\phi)$

$\vec{n} = -\vec{T_\theta}\times\vec{T_\phi}$, $\iint_S (\nabla\times\vec{F})\cdot d\vec{S} = 2\pi$.